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A Generalized Hedonic Pricing Model and Its Applications to the Japanese Real Estate Market*

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1. Introduction

We discuss a generalized hedonic pricing model that incorporates hedonic variables of real estate as well as exogenous variables in an affine form. To this end, we begin our discussion with Ishijima and Maeda (2012, 2015) who developed a unified theoretical model that bridges the gap between the stochastic discounted cash flow model in the finance literature and the hedonic pricing model in the real estate economics literature. Based on the unified pricing model, we discuss a generalized version of hedonic pricing model that incorporates attribute variables of real estate as well as exogenous variables such as interest rates. Furthermore, we show the direction of future empirical analysis which we conduct to help understand the Japanese real estate market in detail.

The rest of this note is organized as follows: Section 2 discusses a generalized hedonic pricing model and in Section 3, we conclude.

2. A Generalized Hedonic Pricing Model

We begin our discussion by reviewing a "theoretical hedonic pricing model" developed by Ishijima and Maeda (2012, 2015). This is an asset pricing model that can simultaneously price real estate and other financial assets. Thereafter, we extend the theoretical hedonic pricing model to incorporate exogenous variables in an affine form.

Proposition 1 (PHD Equations: Ishijima and Maeda 2012, 2015)

Let the occupancy rates $\mathrm{L_t}$ $(\forall t)$ *and dividends yielded by financial securities* $\boldsymbol{D}_t^P$ $(\forall t)$ *be exogenous.*

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Then, financial security prices, real estate prices, and real estate rents are determined by the following equations:

P: Financial asset equilibrium prices (P-equation)

$$\boldsymbol{P}_t = E_t\left[(\boldsymbol{D}_{t+1}^P + \boldsymbol{P}_{t+1})M_{t:t+1}^C\right] \Leftrightarrow \tag{1.1}$$

$$P_{j,t} = E_t\left[\left(D_{j,t+1}^P + P_{j,t+1}\right)M_{t:t+1}^C\right] \ (j = 1, \cdots, N^P) \tag{1.2}$$

H: Real estate equilibrium prices (H-equation)

$$\boldsymbol{H}_t = \boldsymbol{L}_t\boldsymbol{D}_t^H + E_t\left[\boldsymbol{H}_{t+1}M_{t:t+1}^C\right] = \boldsymbol{L}_t\boldsymbol{B}_t\boldsymbol{M}_{t:t}^z + E_t\left[\boldsymbol{H}_{t+1}M_{t:t+1}^C\right] \Leftrightarrow \tag{1.3}$$

$$H_{i,t} = L_{i,t}D_{i,t}^H + E_t\left[H_{i,t+1}M_{t:t+1}^C\right] = L_{i,t}\boldsymbol{b}_{i,t}\boldsymbol{M}_{t:t}^Z + E_t\left[H_{i,t+1}M_{t:t+1}^C\right] \quad (i = 1, \cdots, N^H) \tag{1.4}$$

D: Real estate equilibrium rents (D-equation)

$$\boldsymbol{D}_t^H = \boldsymbol{B}_t\boldsymbol{M}_{t:t}^z \Leftrightarrow \tag{1.5}$$

$$D_{i,t}^H = \boldsymbol{b}_{i,t}\boldsymbol{M}_{t:t}^Z = \sum_{k=1}^{K} b_{ik,t}M_{k,t:t}^Z \ (i = 1, \cdots, N^H) \tag{1.6}$$

where

$$M_{t:t+1}^C = \delta \cdot \frac{\partial u(C_{t+1}, \boldsymbol{Z}_{t+1})/\partial C_{t+1}}{\partial u(C_t, \boldsymbol{Z}_t)/\partial C_t} \tag{1.7}$$

$$\boldsymbol{M}_{t:t}^Z = \frac{\partial u(C_t, \boldsymbol{Z}_t)/\partial \boldsymbol{Z}_t}{\partial u(C_t, \boldsymbol{Z}_t)/\partial C_t} \tag{1.8}$$

$$C_t = \boldsymbol{1}'\boldsymbol{D}_t^P + Y_t \tag{1.9}$$

$$\boldsymbol{Z}_t = \boldsymbol{B}_t'\boldsymbol{L}_t\boldsymbol{1} \tag{1.10}$$

Remark that $M_{t:t+1}^C$ is the intertemporal marginal rate of substitution, u the time-additive utility function of the representative agent, δ the rate of time preference, C_t the general consumption, $\boldsymbol{Z}_t = \left(Z_{1,t} \cdots Z_{K,t}\right)'$ the quantity of K attributes provided by entire real estate, $\boldsymbol{L}_t$ the occupancy rate (i.e. one minus vacancy rate), $\mathbf{B}_t = \left(\boldsymbol{b}_{i,t}\right) = \left(b_{i1,t} \cdots b_{iK,t}\right)$ the quantity of K attributes given by real estate i, $\boldsymbol{M}_{t:t}^Z$ the marginal rate of substitution between K attributes and general consumption.

To interpret the pricing system, the financial asset price is the sum of discounted future dividends as shown in the financial economics literature since Merton (1969) and Lucas (1978). Similarly, the real estate price is the sum of discounted future rents which can be regarded as dividends of financial assets. Moreover, the future rents of real estate can be represented as a linear combination of attribute prices for each of real estate as quoted in the literature of real estate economics or consumer choice since Lancaster (1966, 1971), Rosen (1974), and Ekeland et al. (2004).

We then generate the time series of real estate log prices. Assuming that $\boldsymbol{b}_{i,t+1+\tau} = \boldsymbol{b}_i$ (constant through time), Proposition 1 gives the expected rate of return of real estate.

$$\frac{E_t[H_{i,t+1}] - H_{i,t}}{H_{i,t}} = \frac{\boldsymbol{b}_i}{H_{i,t}} \cdot E_t[\boldsymbol{\kappa}_{i,t+1}] = \boldsymbol{b}_{i,t}^* \cdot E_t[\boldsymbol{\kappa}_{i,t+1}] \tag{1.11}$$

where $\boldsymbol{b}_{i,t}^* := \boldsymbol{b}_i / H_{i,t}$ and

$$\boldsymbol{\kappa}_{i,t+1} := \sum_{\tau=0}^{\infty} (1 - M_{t:t+1}^C) L_{i,t+1+\tau} \boldsymbol{M}_{t+1:t+1+\tau}^Z - L_{i,t} \boldsymbol{M}_{t:t}^Z \tag{1.12}$$

Then the log return of real estate $r_{i,t+1}^H := \log(H_{i,t+1}/H_{i,t}) \approx (H_{i,t+1} - H_{i,t})/H_{i,t}$ is given by

$$r_{i,t+1}^H = \boldsymbol{b}_{i,t}^* \cdot \boldsymbol{\kappa}_{i,t+1} + \varepsilon_{i,t+1} \tag{1.13}$$

where $\varepsilon_{i,t+1}$ has a zero conditional mean. Finally, the log price of real estate i, denoted by $h_{i,t+1}$, is now represented as

$$h_{i,t+1} = h_{i,t} + \boldsymbol{b}_{i,t}^* \cdot \boldsymbol{\kappa}_{i,t+1} + \varepsilon_{i,t+1} \tag{1.14}$$

We remark that the attribute price $\boldsymbol{\kappa}_{i,t+1}$ of Eq. (1.12) is the product of two components that can be interpreted as the cash-flow pricing kernel and hedonic pricing kernel, respectively. The first component is a cash-flow pricing kernel M^C (or stochastic discount factor) which is a marginal rate of substitution between the present and future consumption of nondurable-goods along time horizon. The second component is a hedonic pricing kernel $\boldsymbol{M}^Z$ which is a marginal rate of substitution between the consumption of nondurable-goods and that of real estate attributes at any point in time in the future. In this aspect, our pricing kernel can be regarded as an extension to combine two existing pricing kernels. We can also extend the discussion along discrete-time horizon to provide a stochastic process of real estate prices. On the basis of these theoretical pricing systems, we provide some statistical models that are ready to implement empirical analyses to explore the determinants of real estate prices. That is, our statistical models allow us to incorporate not only hedonic variables of real estate but also exogenous variables $\boldsymbol{z} \in \mathbb{R}^{K_z}$ via the cash-flow pricing kernel. These specifications would help us understand the pricing mechanism of real estate in detail. A possible specification is given by

$$h_{i,t+1} = h_{i,t} + \boldsymbol{b}_{i,t}^*(\boldsymbol{\beta} + \boldsymbol{\nu}_i) + \boldsymbol{\gamma}'\boldsymbol{z} + \varepsilon_{i,t+1} \tag{1.15}$$

We would like to remark that this statistical version of our generalized hedonic pricing model, Eq. (1.15), can be regarded as a typical mixed effects model, if we assume that $\boldsymbol{\nu}_i$ is normal. The mixed effects model has been well-developed in the statistics literature (Fitzmaurice et al. 2006 and McCulloch et al. 2008). Thus, we can easily estimate the generalized hedonic pricing model of Eq. (1.15) by the MIXED procedure in SAS (Littell et al. 2006). By applying the generalized hedonic pricing model, we are now in the position to conduct an empirical analysis to help understand the asset pricing in the Japanese real estate market. We are planning to show the result somewhere in the future.

3. Conclusion

We discussed a generalized hedonic pricing model that incorporates hedonic variables of real estate as well as exogenous variables such as interest rates in an affine form. We then showed the direction of empirical analysis to understand the Japanese real estate market by applying the model developed here.

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